Homework Problem Answers for
***Basic Fluid Mechanics* by David C. Wilcox**

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Chapter 1

1.1 0.211 MJ, 231 kW, 15.0 m/sec
1.3 1.00 Btu, 0.88 ft^3, 120 hp
1.5 8.08 slugs, 117.9 kg, 1.93 m
1.7 29.1 ft/sec^2
1.9 μslug
1.11 psec
1.13 4.85 Pa
1.15 $d/\ell_{mfp} = 413$, yes
1.17 5.1 years
1.19 $2.31 \cdot 10^8$ molecules
1.21 $\rho_w/\rho_a = 904$
1.23 CH_4: 0.71 lb, He: 0.18 lb
1.25 241 kPa
1.27 28.96 slug/slug-mole
1.29 519.6 J/(kg·K), 0.3% larger than the tabulated value
1.31 2770 ft·lb/(slug·$^\circ$R)
1.33 CO_2: 1.52, He: 0.14, H_2: 0.07, CH_4: 0.55, N_2: 0.97, O_2: 1.10
1.35 $SG = 1.014$
1.37 Air: 0.0753 lb/ft^3, Water: 62.4 lb/ft^3
1.39 $c_v = R/(\gamma - 1)$, $c_p = \gamma R/(\gamma - 1)$
1.41 (a) 13.0 kPa, (b) 51° C
1.43 0.27 atm
1.45 (a) 2.36 m, (b) 41.9° C
1.47 He: 0.0005 MPa, Hg: 143 MPa, H_2O: 10.8 MPa
1.49 Air: 0.049 atm, Ether: 302 atm, Glycerin: 2228 atm
1.51 $\Delta\rho/\rho = 0.228$
1.53 -0.02 liter
1.55 (a) $\tau = [n(p + 3040p_a)]^{-1}$, (b) $n = 7.00$
1.57 $\sigma = 0.0045$ lb/ft and $T = 140^\circ$ F
1.59 $D_a/D_w = 1.24$ (24% larger in air)
1.61 $\Delta p_{Hg}/\Delta p_{H_2O} = 4.26$
1.63 $\Delta h = -0.082$ in
1.65 $\Delta h = 5\sigma \cos\phi/(2\rho g d)$
1.67 (a) $\Delta h = 2\sigma/(\rho g r) + (p_a - p_b)/(\rho g)$, (b) $p_b/p_a = 3$

1.69 $\Delta h_2/\Delta h_1 = 0.856$

1.71 $\eta(x) = \sqrt{\sigma/(\rho g)}\,\cot\phi e^{-\sqrt{\rho g/\sigma}\,x}, \quad h = \sqrt{\sigma/(\rho g)}\,\cot\phi$

1.73 (a) $p_a + \frac{1}{4}\rho g s$

(b) Atmospheric pressure from above, $-p_a s^2$, Pressure from below, $p_a s^2 + \frac{1}{4}\rho g s^3$
Weight of the Cube, $-\frac{1}{2}\rho g s^3$, Surface-tension force, $4\sigma s$

(c) $s = 4\sqrt{\sigma/(\rho g)}$

1.75 (a) $\tau_w = \mu U_e/\delta$ (b) $\tau_w = \pi\mu U_e/(2\delta)$ (c) $\tau_w = 3\mu U_e/(2\delta)$

1.77 $\mu_r = 2.12 \cdot 10^{-7}, \quad \omega = 0.65$

1.79 Water: $d\nu/dT < 0$, Air: $d\nu/dT > 0$

1.81 (a) $\mu_e = 0.12/(du/dy)^{1/3}$ (b) $\mu_e/\mu_w \approx 120/(du/dy)^{1/3}$

1.83 $\overline{u} = \frac{1}{2}U, \quad \overline{\frac{1}{2}u^2} = \frac{1}{6}U^2$

1.85 (a) $\mu = 7.50 \cdot 10^{-4}$ slug/(ft·sec) (b) $\rho_{max} = 1.80$ slug/ft^3

1.87 (a) $\mu = m\mu/3$

1.89 $dU/dt = g - 2\pi r\ell\mu g U/[W(R-r)], \quad U \to W(R-r)/(2\pi r\ell\mu)$
Water: 230 ft/sec (not valid), SAE 10W Oil: 6.48 ft/sec (valid),
Glycerin: 0.154 ft/sec (valid)

1.91 $\mu = MAd/(\pi LUD) = 9.66 \cdot 10^{-4}$ kg/(m·sec)

1.93 $Re_h = 0.03$, Couette flow is valid

1.95 $(M+m)dU/dt + \mu AU/h = mg, \quad U \to mgh/(\mu A)$

1.97 $dU/dt = g\sin\alpha - \mu U/(\rho h s), \quad U \to \rho g h s \sin\alpha/\mu = 37.6$ ft/sec

1.99 $(M_1 + M_2)dU/dt = (\sqrt{3}M_1 - M_2)g/2 - \mu(s_1^2/h_1 + s_2^2/h_2)U$

1.101 (a) $Q = \pi(p_1 - p_2)R^4/(8\mu L)$ (b) 0.94 sec

1.103 (a) $u_m = 1.58$ ft/sec (b) $Re_R = 1121$ (c) $F = -4.53 \cdot 10^{-3}$ lb

1.105 $c_f = 4/Re_R$

1.107 $\frac{1}{2}\overline{u}^2 = \frac{1}{6}u_m^2$

Chapter 2

2.9 $[k] = ML/(T^3\Theta)$

2.11 $[\ell] = L$

2.13 $U_{hull} = 2.43\sqrt{L_w}$

2.15 C is dimensionless

2.17 $[\chi] = L^{1/3}/T$

2.27 $u_m = \text{constant} \cdot R^2(dp/dx)/\mu$

2.29 $\nu_t = \text{constant} \cdot k/\omega$

2.31 $\eta = \text{constant} \cdot (\nu^3/\epsilon)^{1/4}$

2.33 $u_m(x) = \text{constant} \cdot \sqrt{J/x}$

2.35 $c = \text{constant} \cdot \sqrt{\sigma/(\rho\lambda)}$

2.37 $Mo = g\mu^4/(\rho\sigma^3)$

2.39 $La = \sigma\rho R/\mu^2$

2.41 2: $\Delta h/\sqrt{A}, \quad \sigma/(\rho g A)$

2.43 2: $\rho^{1/3}\mathcal{R}/M^{1/3},\ \rho^{1/3}M^{2/3}g/\sigma$
2.45 2: $\rho UD/\mu,\ \rho U^2 D/\sigma$
2.47 2: $e/(RT),\ h\nu/(kT)$
2.49 2: $UR/\nu,\ \mathcal{R}/R$
2.51 2: $[\rho c_p/(gk)]^{2/3}c_p\Delta T,\ c_p\mu/k$
2.53 $\Delta p = \rho\,\Omega^2 D^2 f(Q/[\Omega D^3])$
2.55 2: $F\mu_o^2\sigma^2/\rho,\ \mu_o\sigma UL$
2.57 2: $D^2(dp/dx)/(\mu U),\ \rho UD/\mu$
2.59 $\mathcal{T} = \rho\,\Omega^2 d^5\, f(Q/[\Omega d^3])$
2.61 3: $F/(\mu U\ell),\ \rho U\ell/\mu,\ g\ell/U^2$
2.63 3: $U/u_\tau,\ v_w/u_\tau,\ \rho u_\tau k_s/\mu$
2.65 3: $\Delta p/(\omega\mu),\ \ell/D,\ c/D$
2.67 3: $\omega^2 F/(\rho U^4),\ \Omega/\omega,\ \omega L/U$
2.69 3: $\Delta T/T_\infty,\ c_p\mu/k,\ U/\sqrt{c_p T_\infty}$
2.71 3: $D/(\mu\Omega L),\ \rho UL/\mu,\ \Omega L/U$
2.73 3: $\Omega^6 F/(\rho g^4),\ \Omega U/g,\ \Omega^2\ell/g$
2.75 (a) 3 (b) $F/(\rho U^2 d^2),\ \Omega d/U,\ \rho Ud/\mu$ (c) $St = \Omega d/U$
2.77 4: $\rho U_c D/\mu,\ d/D,\ \rho_p/\rho,\ \rho^2 gD^3/\mu^2$
2.79 4: $\omega^2 P/(\rho U^5),\ U/a,\ \omega D/U,\ \rho UD/\mu$
2.81 4: $\Delta h_d/d,\ Q/(Nd^3),\ \nu/(Nd^2),\ g/(N^2 d)$
2.83 4: $T_w/T_\infty,\ c_p/c_v,\ U/a,\ a/\sqrt{c_p T_\infty}$
2.85 $U_m/U_p = 6$
2.87 (a) $U_m = 140$ ft/sec (b) $M_p = 0.188, M_m = 0.029$
2.89 $L_m = 3.30$ ft, $U_m = 18.17$ ft/sec
2.91 $U_m = 630$ mph, $p_m = 3.95$ atm
2.93 (b) $U_p = 55$ ft/sec, $F_p = 175$ lb
2.95 (a) $U_m = 8000$ mph (b) $T_{max} = 5380^\text{o}$ R, model vaporized
2.97 (a) $U_m = 87.4$ m/sec (b) $M_m = 0.239$
2.99 (b) $U_m = 84$ knots
2.101 (a) 3: $\rho LF/\mu^2,\ t/L,\ \rho UL/\mu$ (b) $U_m = 1$ m/sec (c) $F_p = 5000$ N/m
2.103 (a) $R/(\rho gL^3),\ U/\sqrt{gL}$ (b) $U_p = 15$ knots, $R_p = 93750$ lb
2.105 (a) $\mu hU/T,\ w/h,\ \ell/h$ (b) $U_m/U_p = \mu_p/\mu_m$
2.107 (a) $F/(\mu UR),\ r/R,\ \ell/R$ (b) $U_m = \frac{1}{6}U_p$
2.109 (a) $U\tau/h,\ Uh/\nu$ (b) $\tau_m = 1$ sec

Chapter 3

3.1 $\rho_u = 2405$ kg/m^3
3.3 $p = 195$ kPa
3.5 $\rho = 1252$ kg/m^3
3.7 $p = [(\alpha+1)p_a + \frac{6}{7}\rho_a gz]^{7/6}/[(\alpha+1)p_a]^{1/6} - \alpha p_a$
3.9 $p = p_a + 28\rho gh$

3.11 $h = 8.17$ m
3.13 450 m
3.15 $p = 0.303$ atm, $T = -44^o$ C
3.17 $p = 2.44$ psi, $T = -69.7^o$ F
3.19 Power = 256 hp
3.21 $\alpha = (\gamma - 1)g/(\gamma R)$
3.23 $\tilde{\rho} = 0.15\rho$
3.25 $W = 7865$ lb
3.27 $\Delta z = 1.0$ ft
3.29 (a) $p_1 - p_2 = (\rho_2 h_2 - \rho_1 h_1 - \rho_{Hg}\lambda)g$ (b) $p_1 - p_2 = -8.9$ kPa
3.31 $\Delta z = 1.14$ m
3.33 $p_A = p_a + 29.94\rho g h$
3.35 $\alpha = 39.9^o$
3.37 (a) $h = (p_1 - p_2)/(g\Delta\rho)$ (b) $h_{Kerosene} = 48.9$ mm, $h_{Oil} = 15.1$ mm
3.41 $I = 5\pi R^4/2$
3.43 $H = 3h/2$
3.45 $h_p = 11h/24$
3.47 $h = H/3$
3.49 $z_{cp}(R)/z_{cp}(0) = 17/10$
3.51 $F_h = (2 - z_{cp}/L)\rho g \overline{z} A$ (a) $F_h = \frac{33}{32}\pi\rho g L^3$ (b) $F_h = \rho g L^3$
3.53 $(1 + \epsilon)(h/H)^2(4 - h/H) = 3$
3.55 $F_s = \frac{\pi}{4}\rho g R^3$
3.57 $F = 141$ tons, $h_{cp} = 8$ ft
3.59 $h = 2.18s$
3.61 $\mathbf{F} = \rho g h^3(3\mathbf{i} + 4\mathbf{k})$, $z_{cp} = 2h/3$
3.63 $\mathbf{F} = -2\rho g h^3(18\mathbf{i} - 23\mathbf{k})$, $z_{cp} = 2h$
3.65 $\mathbf{F} = 1.25\rho g h^3(2\mathbf{i} + \pi\mathbf{k})$, $z_{cp} = 2h/3$
3.67 $\mathbf{F} = 3\rho g H^3(\mathbf{i} + \mathbf{k})$
3.69 $\rho_2/\rho_1 = 2$, $F_x = -\rho_1 g H^3/6$
3.71 $\mathbf{F} = -3.5\rho g H^3(\mathbf{i} + \mathbf{k})$
3.73 (a) $\mathbf{F} = -\rho g \ell^3[3\mathbf{i} + (4 + \pi)\mathbf{k}]$
(b) $z_{cp} = 14\ell/9$
(c) $x_{cp} = 10\ell/[3(4 + \pi)]$
3.75 $F_{buoy} = 0.25\rho g V$
3.77 $V = 1839$ cm^3, $\rho = 2828$ kg/m^3
3.79 35 cm^3
3.81 29% gold
3.83 $\ell = 1.95L$
3.85 $T = 3.67$ kN
3.87 $\mathbf{F} = 2\rho g H^3\mathbf{i}$
3.89 $F = 1.437\rho_a g h^3$, $z_{cp} = 0.696h$
3.91 Weight = $(1 + \frac{3}{4}\pi)\rho g R^2 L$, Force = $\frac{1}{2}\rho g R^2 L$
3.93 $\mathbf{F} = (3/16)\rho g D^2 w(2\mathbf{i} - \pi\mathbf{k})$
3.95 $\rho_b = 0.42\rho$

3.97 (a) $F_L = 16\rho g h^3$ (to the right), $F_R = 16\rho g h^3$ (to the left), $T = 2\rho g \ell^3$ (to the right)
(b) $F_S = \frac{2}{3}\rho g h^3 - \rho g \ell^3$
3.99 (a) $F_L = 6\rho g h^3$ (to the right), $F_R = 6\rho g h^3$ (to the left), $T = \frac{3}{2}\rho g \ell^3$ (to the right)
(b) $F_S = \frac{1}{6}\rho g h^3 - \frac{3}{4}\rho g \ell^3$
3.101 $N = 4/3$
3.103 $\rho_b = \frac{5}{8}\rho$
3.105 $\rho_c = 5\rho$
3.107 (a) $F_H = T - \frac{5}{8}\rho g H^3$ (b) $F_H = -\frac{49}{120}\rho g H^3$, $T = \frac{13}{60}\rho g H^3$ (c) $\lambda = 14$

Chapter 4

4.1 $\mathbf{a} = \mathbf{0}$
4.3 Unsteady: $\partial u/\partial t = -\tau^{-1}[U_o - (x/\tau)]e^{-t/\tau}$
Convective: $u\partial u/\partial x = -\tau^{-1}[U_o - (x/\tau)]e^{-2t/\tau}$
Total: $\mathbf{a} = -\tau^{-1}[U_o - (x/\tau)](1 + e^{-t/\tau})e^{-t/\tau}\,\mathbf{i}$
4.5 (a) $\mathbf{a} = (\dot{A} + A^2)(x\,\mathbf{i} + y\,\mathbf{j})$ (b) $\mathbf{a} = (\dot{A}+2A^2x)x^2\mathbf{i}-(\dot{A}-2A^2y)y^2\mathbf{j}$
4.7 (a) $\mathbf{a} = (2U^2x^2/H^4)(x\,\mathbf{i} + 3y\,\mathbf{j} + 6z\,\mathbf{k})$ (b) $\mathbf{a} = (U^2/H^2)(4x\,\mathbf{i} + 9y\,\mathbf{j} + z\,\mathbf{k})$
4.9 $\mathbf{a} = \mathbf{0}$
4.11 $\mathbf{a} = \dot{U}\,\mathbf{i} + \lambda^{-1}(x\dot{U} + U^2)\,\mathbf{j})$
4.13 $\mathbf{a} = (U/H)^2(x\,\mathbf{i} + y\,\mathbf{j})$
4.15 (a) $\mathbf{a} = -\Gamma^2/(4\pi^2 r^3)\,\mathbf{e}_r$ (b) $\mathbf{a} = -\mathcal{D}^2/(2\pi^2 r^5)\,\mathbf{e}_r$
4.17 $x = L[(1 + x_o/L)e^{Ut/L} - 1]$, $y = y_o e^{-Ut/L}$
4.19 $\mathbf{r} = x_o e^{2Ut/R}\mathbf{i} + y_o e^{-2Ut/R}\mathbf{j}$
4.21 $r = \sqrt{r_o^2 + Qt/\pi}$, $\theta = \theta_o$
4.23 $r = r_o$, $\theta = \theta_o + \Omega t$
4.25 (a) $\boldsymbol{\omega} = \mathbf{0}$ (b) $\boldsymbol{\omega} = (du_y/dx - du_x/dy)\,\mathbf{k}$ (c) $\boldsymbol{\omega} = -(U/\delta)e^{-y/\delta}\mathbf{k}$
4.27 $\boldsymbol{\omega} = \mathbf{0}$ for all values of A
4.29 (a) and (b) $\nabla \times \mathbf{u} \neq \mathbf{0} \implies$ rotational
4.31 (a), (b) and (c) $\boldsymbol{\omega} = \mathbf{0}$
4.33 $\boldsymbol{\omega} = (-z\sin\theta\mathbf{e}_r + 3z\cos\theta\mathbf{e}_\theta + r\sin\theta\mathbf{k})/r^4$
4.35 $\mathbf{a} = Uy/(2\sqrt{\pi\nu}\,t^{3/2})\exp[-y^2/(4\nu t)]\mathbf{i}$, $\boldsymbol{\omega} = (U/\sqrt{\pi\nu t})\exp[-y^2/(4\nu t)]\mathbf{k}$
4.37 $\Gamma = 8\Omega HL \neq 0$
4.41 $\Gamma = -UL^2/H$
4.43 $y^2 - x^2 =$ constant
4.45 $y/x =$ constant
4.47 $x^2 y =$ constant
4.49 $r/\cos^2\theta =$ constant
4.51 $(r - R^2/r)\sin\theta =$ constant
4.53 (a) $y - k^{-1}\sin[k(x - Ut)] =$ constant (b) $y = (x - x_o)\cos(kx_o)$
4.55 $\dot{m} = 8n\rho U_{max}h^2/(n + 1)$, $\dot{m}_{n=7}/\dot{m}_{n=1} = 7/4$
4.57 $\dot{m} = 2\rho UA$
4.59 $\dot{P}_z = -0.25\rho U^2 h^2$

4.61 $\dot{m} = 0$
4.63 $dN_R/dt = -0.1$ person/sec
4.65 6.0 ft/sec
4.67 $dN/dt = -\dot{N}_{pop} - nUA$
4.69 $dN_{truck}/dt = -0.2\rho UA$
4.71 $dN_c/dt = -dN_g/dt - (U_g A_g + U_c A_c)/V$

Chapter 5

5.1 (a) $u_r(r, \theta, z) = f(\theta, z)/r$ (b) $u_R(R, \theta, \phi) = g(\theta, \phi)/R^2$
5.3 $\rho(x) = \rho_a(1 + x/x_o)^{-1}, \quad x = \frac{3}{2}x_o$
5.5 $u(x, t) = u_o - (\rho_a/\rho - 1)x/\tau$
5.7 No, Yes
5.9 Yes, Yes
5.11 Yes, No
5.13 $\Lambda = -1, \quad \omega$ is arbitrary
5.15 $v(x, y) = \frac{1}{2}C(x^2 - y^2) + \text{constant}$
5.17 $u(x, y) = -Dy/(x^2 + y^2) + \text{constant}$
5.19 Yes, Yes
5.21 $u_\theta(r, \theta) = 3Ar^2 \cos\theta + \text{constant}/r$
5.23 $p(x, y, z, t) = -\rho g z + f(t)$
5.25 $p(x, y, z, t) = (\rho U y/\tau)e^{-t/\tau} + f(t)$
5.27 (a) Yes (b) Yes (c) $p(x, y) = p_t - \frac{1}{2}\rho A^2(x^2 + y^2)$
5.29 $\ell_1 = L_2 - \frac{2}{5}\Omega^2 L_3^2/g, \quad \ell_2 = L_2 + \frac{1}{10}\Omega^2 L_3^2/g$
5.31 $\lambda = 0.57$
5.33 $\Omega = 2\sqrt{gh}/R$
5.35 $h_{min} = h - \Omega^2 R^2/(4g), \quad h_{max} = h + \Omega^2 R^2/(4g)$
5.37 $p_{max} = p_a + \rho g h \cos\alpha$ at $z = 0$
5.39 Ω has opposite sign for observers at the north and south poles
5.41 $h_o = h - a\ell/(2g)$
5.43 $U_P = 3$ m/sec
5.45 $p_B - p_C = 24$ kPa
5.47 $\partial\mathbf{u}/\partial t + \mathbf{u} \cdot \nabla\,\mathbf{u} = -\nabla p/\rho - \boldsymbol{\Omega} \times \boldsymbol{\Omega} \times \mathbf{r} - 2\boldsymbol{\Omega} \times \mathbf{u} - \Omega^2 Ut\,\mathbf{i} + 2\Omega U\,\mathbf{j}$, Not invariant
5.49 $p - p_a = \frac{1}{2}\rho(2gh - U^2) = 37.57$ kPa
5.51 $p_b = p_a + 1.25\rho g h = 2.21$ atm, $U = 21.0$ m/sec
5.53 $p_2 - p_1 = 0.25$ psi
5.55 $U = 3.58$ ft/sec, $\Delta p = -1.07 \cdot 10^{-4}$ psi
5.57 70 mph: $p_{max} = 1.006$ atm, 200 mph: $p_{max} = 1.048$ atm
5.59 $p_i - p_a = 2\rho g h - \frac{1}{2}\rho U_i^2 = 367$ psf
5.61 (a) $U = \sqrt{2gh_1}, \quad p_{min} = p_a - \rho g(h_1 + h_2)$ (b) $U = 8.86$ m/sec, $p_{min} = 0.223$ atm
5.63 $d = D[1 + \pi^2\rho^2 g D^4(\ell - z)/(8\dot{m}^2)]^{-1/4}$
5.65 $p_t = 1739$ psf, $U = 468$ ft/sec
5.67 $Q_{min} = 0.046\ \text{m}^3$/sec

Chapter 6

6.1 Easy: $\mathbf{n} = \mathbf{i}\cos\phi - \mathbf{j}\sin\phi, \quad \mathbf{u}\cdot\mathbf{n} = U,$

$\iint \rho u(\mathbf{u}\cdot\mathbf{n})dA = \rho U^2 A\cos\phi, \quad \iint \rho v(\mathbf{u}\cdot\mathbf{n})dA = -\rho U^2 A\sin\phi$

Hard: $\mathbf{n} = \mathbf{i}, \quad \mathbf{u}\cdot\mathbf{n} = U\cos\phi,$

$\iint \rho u(\mathbf{u}\cdot\mathbf{n})dA = \rho U^2 A\cos\phi, \quad \iint \rho v(\mathbf{u}\cdot\mathbf{n})dA = -\rho U^2 A\sin\phi$

6.3 $\mathbf{n}_1 = -\mathbf{i}, \;\; \mathbf{n}_2 = \frac{1}{2}\sqrt{2}(\mathbf{i}+\mathbf{j}), \;\; \mathbf{n}_3 = \frac{1}{2}(\sqrt{3}\,\mathbf{i}-\mathbf{j})$

$\mathbf{u}_1\cdot\mathbf{n}_1 = -U_1, \;\; \mathbf{u}_2\cdot\mathbf{n}_2 = U_2, \;\; \mathbf{u}_3\cdot\mathbf{n}_3 = U_3$

$\mathbf{u}_1\cdot\mathbf{n}_1 A_1 = -\frac{9}{4}\pi U_1 d^2, \;\; \mathbf{u}_2\cdot\mathbf{n}_2 A_2 = \pi U_2 d^2, \;\; \mathbf{u}_3\cdot\mathbf{n}_3 = \frac{1}{4}\pi U_3 d^2$

$\mathbf{F} = (1.824\,\mathbf{i} - 0.312\,\mathbf{j})\pi d^2 \Delta p$

6.5 Inlet: $A = 4h^2, \;\; \mathbf{n} = -\mathbf{i}\cos\beta - \mathbf{j}\sin\beta, \quad \mathbf{u}\cdot\mathbf{n} = -U, \quad \mathbf{u}\cdot\mathbf{n}A = -4h^2 U$

Outlet: $A = 4h^2\csc\beta, \;\; \mathbf{n} = \mathbf{j}, \quad \mathbf{u}\cdot\mathbf{n} = U\sin\beta, \quad \mathbf{u}\cdot\mathbf{n}A = 4h^2 U$

6.7 Inlet: $\mathbf{n} = -\mathbf{i}, \;\; \mathbf{u}\cdot\mathbf{n} = -U, \;\; \iint \rho u(\mathbf{u}\cdot\mathbf{n})dA = -15\rho U^2 H^2, \;\; \iint \rho v(\mathbf{u}\cdot\mathbf{n})dA = 0$

Upper Outlet: $\mathbf{n} = \mathbf{j}, \quad \mathbf{u}\cdot\mathbf{n} = \frac{4}{3}U, \;\; \iint \rho u(\mathbf{u}\cdot\mathbf{n})dA = 0, \;\; \iint \rho v(\mathbf{u}\cdot\mathbf{n})dA = \frac{40}{3}\rho U^2 H^2$

Lower Outlet: $\mathbf{n} = -\mathbf{j}, \quad \mathbf{u}\cdot\mathbf{n} = \frac{5}{9}U, \;\; \iint \rho u(\mathbf{u}\cdot\mathbf{n})dA = 0, \;\; \iint \rho u(\mathbf{u}\cdot\mathbf{n})dA = -\frac{25}{9}\rho U^2 H^2$

6.9 $\mathbf{F} = -\frac{1}{2}p_a A_i\,\mathbf{i}$

6.11 $\alpha = (100 - 225\beta)/64$; for $\beta = 3/5$, fluid enters Outlet B with speed $\frac{35}{64}U$

6.13 $dh/dt = -0.06U(d/D)^2$, the tank is emptying

6.15 $U = 1.0$ m/sec, $u = 11.2$ m/sec

6.17 $U_v = 3U/16$

6.19 $U_2 = 8U_1/3$

6.21 $V = -6U$

6.23 $\mathbf{u}_{avg} = 1.04\,U\mathbf{k}$ for an observer on the sphere

6.25 $A_{cs} = A/10$

6.27 $D = 0.208H$

6.29 $dh/dt = -U/100$

6.31 $U_r = \frac{1}{2}Ur/\ell$

6.33 (b) $p_c = 80532$ psf

6.35 $z = 76.4d$

6.37 $\beta = \frac{1}{4}\alpha$

6.39 $dM/dt = -2.95\rho_a V_o r_o h$

6.41 $u_{max} = 1.5U_o$

6.43 $p_1 - p_2 = \frac{24}{25}\rho U^2$

6.45 (a) $(U_2 - U_1)H = 2Vh$

(b) $(U_2^2 - U_1^2) - (hV^2/H)\cos\phi = (p_1 - p_2)/\rho$

(c) $U_2 = U_1(H + 4h)/(H - 4h)$

6.47 $U_1 = 0.32U, \;\; (p_4 - p_1) = 0.035(p_2 - p_a) + 0.135\rho U^2$

6.49 $\phi = \sin^{-1}(1/3) = 19.5^{\circ}, \;\; \Delta p = 0.707\rho U^2$

6.51 $\mathbf{F} = 0.25\pi\rho(U_j - U)^2 d^2\,\mathbf{i}, \;\; |\mathbf{F}| = 11.3$ N

6.53 $\mathbf{F} = 0.18\pi\rho U_i^2 d^2\,\mathbf{i}$

6.55 $\tau = \frac{1}{16}D, \;\; \mathbf{F} = \frac{1}{2}\pi\rho U^2 D^2\,\mathbf{i}$

6.57 Solution 1: $V = 0.25U_i,\;\; U_o = 0.625U_i,\;\; F_y = -0.027\rho U_i^2 A$
Solution 2: $V = 0.75U_i,\;\; U_o = -0.125U_i,\;\; F_y = -0.244\rho U_i^2 A$
6.59 $V = \frac{4}{3}U,\;\; \Delta p = \frac{4}{9}\rho U^2,\;\; R_x = -\frac{2}{9}\rho U^2 A\cos\phi$
6.61 $\Delta p = -0.48\rho U^2$
6.63 $\phi = 18.4^o$
6.65 $U_b = U/3$
6.67 $U_j = 4.36\; U_o$
6.69 $U_o = \frac{1}{2}U_i,\;\; R_y/R_x = 5/(2+5\cos\phi)$
6.71 $\mathbf{R} = -\dot{m}[(U_1 + U_2\cos\beta)\,\mathbf{i} - U_2\sin\beta\,\mathbf{j}]$
6.73 $V = 4U,\;\; \mathbf{R} = \pi\rho U^2 d^2(\sqrt{2}\,\mathbf{i} + \mathbf{j} + \mathbf{k})/\sqrt{2}$
6.75 $V = U,\;\; \mathbf{F} = 1.1\rho U^2 A[(16/55 + \cos\phi)\,\mathbf{i} + (2/11 - \sin\phi)\mathbf{j}]$
6.77 $\mathbf{F} = 1.1\pi\rho U^2 d^2\,\mathbf{i}$
6.79 $R = -0.54\pi\rho U_i^2 A$
6.81 $\Delta p = -0.44\rho U^2$
6.83 $\mathbf{R} = 1.125\pi\rho U^2 D^2\,\mathbf{i}$
6.85 (a) $V = \sqrt{2gh}$ (b) $\mathbf{F} = \frac{1}{2}\pi\rho g h d^2\,\mathbf{i}$ (c) $h = 428d$
6.87 $\mathbf{F} = \rho U^2 A[(1 + 2.5\cos\phi)\mathbf{i} - (1 - 2.5\sin\phi)\,\mathbf{j}]$
6.89 $\mathbf{F} = \frac{1}{32}\pi\rho V^2 D^2(-3\,\mathbf{i} + 2\,\mathbf{j})$
6.91 (a) $v = 2U$ (b) $U = 2\sqrt{gh/3}$
6.93 $\Delta p = 0.079\rho U_1^2$, Bernoulli's equation is not satisfied
6.95 $U_j = 5U,\;\; \Delta p_1 = 12\rho U^2,\;\; \Delta p_2 = 12.18\rho U^2,\;\; \mathbf{F} = 24.82\rho U^2 A\,\mathbf{i}$
6.97 (a) $\mathbf{F}_{wall} = -\rho U^2 H\sin\phi\,\mathbf{j}$ (b) $U_1 = \frac{1}{2}(H/h - \cos\phi)U,\;\; U_2 = \frac{1}{2}(H/h + \cos\phi)U$
(c) $U_1 = U_2$ as required by symmetry
6.99 (a) $Mg = \frac{1}{4}\pi\rho W^2 d_o^2(1 - \frac{4}{3}gh/W^2 - \sqrt{1 - 2gh/W^2}\cos\phi)$ (b) $Mg = 0.467$ kN
6.101 (a) $Mg = \frac{1}{4}\pi\rho W^2 d_o^2(1 + \sqrt{1 - 2gh/W^2})$ (b) $Mg = 6.18$ kN
6.103 $\mathbf{F} = -\frac{1}{4}\pi\rho U^2(D^2 - d^2)(1 + \sin\phi)\,\mathbf{i}$
6.105 $p_1 - p_2 = \frac{17}{15}\rho U_\infty^2$
6.107 (a) $R_x = -0.719\pi\rho U^2 D^2$ (b) $R_x = -1.052\pi\rho U^2 D^2$ (46% larger)
6.109 $K = \frac{8}{7}U$
6.113 $v_{min} = \frac{2}{3}U,\; C_p = -\frac{5}{9}$
6.115 (a) $dM/dt + \rho u_e A = 0,\;\; d(MU)/dt + \rho(U - u_e)u_e A = -\mu_s Mg$
(b) $U(t) = u_e \ell n(M_o/M) - \mu_s gt$
6.117 $M(t) = M_o\exp(-\sqrt{\rho_e A_e}\,Pt),\;\; V_s(t) = V_{so}$
6.119 (a) $dM/dt + \frac{1}{4}\pi\rho u_e d^2 = 0,\;\; d(MU)/dt + \rho_e(U - u_e)(\frac{1}{4}\pi u_e d^2) = -\rho U^2 A C_D$
(c) $U = 3.38$ knots
6.121 (a) $dm/dt = \rho(U_j - U)A,\;\; mdU/dt = \rho(U_j - U)^2 A - \mu mg$
(b) $mdU/dm = U_j - U$ (c) $U(t) = \frac{4}{5}U_j$

Chapter 7

7.1 (a) $n = 1.22$ (b) $W = -2.544\cdot 10^6$ J (c) $Q = -1.145\cdot 10^6$ J
7.3 $T_2 = 59.8^o$ C, $W_c = 3.01$ MJ

7.5 $s = -R\ell np + \text{constant}$
7.7 $\tau_s = 1/(\gamma p)$, smaller than the isothermal value by a factor of $1/\gamma$
7.13 $\mathbf{R} = -\frac{4}{5}\sqrt{\dot{m}\dot{W}_s}\,\mathbf{i}$
7.15 $P = \dot{m}[c_v\Delta T + 3p/\rho - \frac{15}{2}\dot{m}^2/(\pi^2\rho^2 d^4)]$
7.17 $u_2 = 83.3$ m/sec, $T_2 = 18.5^\circ$ C
7.19 $T_f = 19.5^\circ$ C
7.21 $\dot{Q} = 240$ Btu/sec, No
7.23 $\alpha_1 - \alpha_2 = 0.04$
7.25 $\alpha = (n+1)^3/(3n+1)$, $\alpha = 1.059, 1.045, 1.036$ for $n = \frac{1}{6}, \frac{1}{7}, \frac{1}{8}$
7.27 $h_p = 0.07U^2/(2g)$
7.29 (a) Laminar, $f = 0.032$ (b) Turbulent, $f = 0.026$ (c) $k_s/D \approx 0.006$
7.31 Glass: $f = 0.0090$, $h_L = 31.7$ m
Wrought iron: $f = 0.0137$, $h_L = 48.3$ m
Cast iron: $f = 0.0199$, $h_L = 70.1$ m
7.33 $\overline{u} = 4.48$ ft/sec
7.35 Laminar: $h_L = 0.38$ in, Turbulent: $h_L = 6.23$ in
7.37 (a) $\overline{u} = \sqrt{2(p_s - p)/[\rho(1 - fL/D)]}$ (b) $\overline{u} = \overline{u}_a/\sqrt{1 - fL/D}$ (c) $L/D = 3.02$
7.39 $U_2 = 4U$, $h_p = 9.5U^2/g$
7.41 $2gh_L/U^2 = 2.2g\Delta z/U^2 - 1.05 = 0.52$
7.43 $h_p = 2\Delta z + 1.75U^2/g = 12.42$ m
7.45 $h_p = \Delta z + 1600\dot{m}^2/(\pi^2\rho^2 g D^4)$
7.47 $\dot{W}_t = 0.815\dot{m}gH$
7.49 $Fr_H = 1/3$
7.51 $\dot{W}_t = \frac{5}{6}\dot{m}gH = 8385$ kW
7.53 $h = 1.36d$
7.55 $D_h = (1 - \alpha)D$
7.57 (a) $f = 1.25gD/U^2 - 0.012$ (b) $k_s = 0.0002D$
7.59 (a) $D = 0.01056L/(2g\Delta z/U^2 - 1)$ (b) $D = 9.0$ cm
(c) $D = 0.01056L/(2g\Delta z/U^2 - 1.3848)$, $D = 9.1$ cm
7.61 $h_L = 0.75\overline{u}^2/(2g)$
7.63 $U = \frac{2}{3}\sqrt{gL}$
7.65 $gh_L/U^2 = 9044, 224, 28, 9$
7.67 (b) $\overline{u}_1 = 7.45$ ft/sec, $\overline{u}_2 = 6.72$ ft/sec $\overline{u}_3 = 5.64$ ft/sec
(c) $Re_{D_1} = 1.15 \cdot 10^5$, $Re_{D_2} = 1.56 \cdot 10^5$ $Re_{D_3} = 1.74 \cdot 10^5$
7.69 $Re_D = 3.21 \cdot 10^7$
7.71 $p_1 - p_2 = 1018$ kPa
7.73 $W = 2\sqrt{gH}$
7.75 The company will earn 1 cent/kilowatt-hour and, if well managed, will be profitable.
7.77 Chézy-Manning: $\overline{u} = 8.73$ ft/sec, Colebrook: $\overline{u} = 8.54$ ft/sec
7.79 $n = 0.033$
7.81 Brickwork: $Q = 46.1$ m^3/sec, $Fr = 0.70$ Stony: $Q = 19.7$ m^3/sec, $Fr = 0.30$
7.83 $Fr = 0.138$
7.85 (b) $A = y^2$, $\alpha = 45^\circ$
7.87 $\overline{u} = 2$ ft/sec, $y = 9.3$ in

7.89 $\overline{u}_{min} = 8.86$ m/sec, $\overline{u}_{down} = 17.72$ m/sec
7.91 $U\Delta t/\ell = 2Fr^2/(1 - Fr^2)$, $\Delta t \to \infty$ as $Fr \to 1$
7.93 $Fr_1 = 0.042$
7.95 $y_c = 4.82$ ft, $S_c = 0.0132$, $\theta_c = 0.75^\circ$
7.97 Supercritical, $E/E_{min} = 1.52$
7.99 (a) $\overline{u}_0 = y_1\sqrt{2g/(y_0 + y_1)}$, $\overline{u}_1 = y_0\sqrt{2g/(y_0 + y_1)}$, $Fr_1 = y_0\sqrt{2g/[y_1(y_0 + y_1)]}$
(b) $Fr_1 = 2.26$ (c) $y_2 = 8.20$ ft
7.101 $y_2 = 6.79$ ft, $Fr_1 = 2.51$, $Fr_2 = 0.46$, $\overline{u}_1 = 21.10$ ft/sec, $\overline{u}_2 = 6.84$ ft/sec
7.103 $y_2 = 1.68$ m, $h_L = 49$ cm

Chapter 8

8.1 Air: $U = 982$ km/hr, Water: $U = 4266$ km/hr
8.3 (a) $M = 1.16$, transonic (b) $M = 0.24$, incompressible (c) $M = 2.02$, supersonic
(d) $M = 0.95$, transonic (e) $M = 5.24$, hypersonic
8.5 $U_{LA} = 919$ ft/sec, $U_{Denver} = 903$ ft/sec, Faster in Los Angeles
8.7 $T_a = 51^\circ$ F
8.9 (a) $\gamma = 1.343, 1.323, 1.290, 1.286$ (b) $T = 365^\circ$ C
8.11 $a/a_o = 0.08, 0.06, 0.05$
8.13 $M = 0.029, 0.148, 0.298$
8.15 Air: $p_t = 364$ kPa, $T_t = 136^\circ$ C Methane: $p_t = 243$ kPa, $T_t = 81^\circ$ C
8.17 Air: $T = -416^\circ$ F, Helium: $T = -433^\circ$ F
8.19 $p_t = 4.25$ atm, $U = 447$ m/sec
8.21 $p_A/p_B = 42.91$, $T_A/T_B = 2.927$, $T_{tA}/T_{tB} = 1$
8.23 (a) $M_\infty = 0.288$, $\Delta T/T_\infty = -1.6\%$ (b) $M_\infty = 0.576$, $\Delta T/T_\infty = -6.4\%$
8.25 (a) $M = 0.571$ (b) $M = 0.958$ (c) $M = 0.257$
8.27 (a) Helium (b) $T = 28^\circ$ C (c) $p = 72$ kPa
8.29 $M = 3$
8.31 (a) $u_2 = 887$ m/sec (b) $p_2 = 474$ kPa (c) $p_2 = 397$ kPa
8.33 $M_1 = \sqrt{13} = 3.6$
8.35 Air: $M_2 = 0.475$, $T_{t2} = 1540^\circ$ R, $p_2 = 217$ psi
He: $M_2 = 0.522$, $T_{t2} = 2200^\circ$ R, $p_2 = 231$ psi
8.37 $p_2 = 79.0$ kPa
8.39 $M_1 = 1.32$
8.41 Air: $M_1 = 1.5$, $T_1 = 0^\circ$ C He: $M_1 = 1.55$, $T_1 = -40^\circ$ C
8.43 Air: $p_t = 1.49$ atm CO_2: $p_t = 1.46$ atm
8.45 $M_1^2 = [1 + \frac{1}{2}(\gamma - 1)M_2^2]/[\gamma M_2^2 - \frac{1}{2}(\gamma - 1)]$
8.47 $M_2 = 1.0000, 0.9474, 0.8888, 0.7483$ Error $= 0\%, 0.6\%, 2.5\%, 11.1\%$
8.53 $A_{\text{throat}} = 20$ cm^2
8.55 $p^*/p_t = 0.544$
8.57 $p^*/p_t = 0.487$
8.59 $p_2 = 308$ kPa

8.61 Yes, because the nozzle is overexpanded
8.63 $M_e = 0.271$
8.65 $M_e = 0.112$
8.69 $A/A^* = \frac{1}{2}[2(2\gamma-1)/(\gamma+1)]^{(\gamma+1)/[2(\gamma-1)]}$
$A/A^* \to \infty$ for $\gamma \to 1$, $A/A^* \to 1$ for $\gamma \to \infty$
8.71 $M = 0.57$ or $M = 2.02$
8.73 Cadillac: Error $= -0.2\%$ SST: Error $= -80\%$
8.75 $M_1 = 3.53$, $T_t = 27^{\circ}$ C, $p_{t1} = 221$ kPa
8.77 $M_3 = 0.22$
8.79 (b) $U_s = 18906$ ft/sec
8.81 (b) $p_t = 128$ kPa, $M_e = 0.595$ (c) $F = 298$ N
8.83 $M_2^2 = 1/M_1^2$, $\rho_2/\rho_1 = M_1^2$, $p_2/p_1 = M_1^2$, $T_2/T_1 = 1$

Chapter 9

9.1 (a) $P = \dot{m}\Omega\ell(w_e \cos\phi - \Omega\ell)$ (b) $w_e = 20$ m/sec (c) $\phi = 60^{\circ}$
9.3 $\beta_2 = \tan^{-1}[(r_1/r_2)^2 \tan\beta_1]$
9.5 (b) $p_2 - p_1 = 306$ kPa
9.9 $Q = 4070$ gal/min, $\dot{W}_p = 612$ hp
9.11 (a) $Q = 151.9$ ft^3/sec, $\dot{W}_p = 2.41$ hp (b) $\alpha_2 = 67.7^{\circ}$
9.13 $Q = 29.45$ ft^3/sec, $\dot{W}_p = 855$ hp
9.15 $Q = 48.4$ ft^3/sec, $\dot{W}_p = 18.3$ hp
9.17 $\beta_1 = 6.9^{\circ}$, $\beta_2 = 3.4^{\circ}$
9.19 Water: $h_p = 5/6$ ft, Air: $h_p = 691$ ft, Helium: $h_p = 5005$ ft
9.21 $[\tilde{N}_s] = L^{3/4}/T^{3/2}$, $\tilde{N}_s = 2733 N_s$
9.23 $Q_o < Q < 5Q_o$
9.25 $Q^* = 1960$ gal/min, $\eta_p = 0.81$
9.27 (a) $\beta_2 = 127^{\circ}$, $r_2 = 5.00$ in, $b_2 = 1.00$ in
9.29 NPSH $= (p_a - p_v)/(\rho g) - \frac{7}{5}H$
20° C: no cavitation
70° C: cavitation
9.31 $H_{\max} = \frac{8}{11}(p_a - p_v)/(\rho g)$
9.33 $C_{Q^*} = 0.0148$, $C_{H^*} = 0.151$, $C_{P^*} = 0.00254$, $\eta_p = 0.88$, $N_s = 0.50$
9.35 $\Omega_{min} = 1549$ rpm
9.37 $C_{Q^*} = 0.200$, $C_{H^*} = 0.0789$, $C_{P^*} = 0.0208$, $\eta_p = 0.76$, $N_s = 3.0$
9.39 $\eta_2 = 0.76$
9.41 $N_s = 0.084$, Single-jet Pelton wheel
9.43 $N_s = 1.49$, Francis turbine
9.45 $N_s = 4.22$, Kaplan turbine
9.47 $\Omega = 135$ rpm
9.49 (a) $\tau_{max} = \frac{9}{8}\rho Q V_j r$ at $\phi = 120^{\circ}$ (b) $\tau_{max} = \rho Q V_j r$ at $\phi = 180^{\circ}$

Chapter 10

10.1 (c) Irrotational flow is isentropic, isentropic flow is irrotational
10.3 (c) $d\tilde{\boldsymbol{\omega}}/dt = \tilde{\boldsymbol{\omega}} \cdot \nabla \mathbf{u}$ where $\tilde{\boldsymbol{\omega}} \equiv \boldsymbol{\omega}/\rho$
10.5 $\Gamma = 1.106 \cdot 10^5$ ft²/sec, $\Omega = 11.7$ rpm
10.7 $U = 143$ mph
10.9 $\Gamma_a = U \Delta x$
10.11 (a) $\Gamma = -UL$ (b) $\Gamma = -UL$
10.13 $\tau_w = -\mu \omega_w$
10.15 $\delta = 3.2$ mm
10.17 $\delta \approx 3.2$ in, $\tan^{-1}(\delta/x) = 0.6^{\circ}$
10.19 (a) $\partial u/\partial x \sim \frac{1}{6}(u_o/L)(x/L)^{-5/6}$
(b) $\partial u/\partial y \sim \frac{1}{6} u_o \sqrt{u_o/(\nu L)}\,(x/L)^{-1/4}$
(c) $(\partial u/\partial y)/(\partial u/\partial x)|_{x=L} \sim \sqrt{Re} = 100, 316, 1000$ for $Re = 10^4, 10^5, 10^6$
10.21 $\rho_c = \rho[1 + U^2 C_D/(2gs)] = 11.29$ slug/ft³
10.25 200 km
10.27 Configuration (a): $U = 49.4$ ft/set, Configuration (b): $U = 37.5$ ft/sec
10.29 (a) 4.0 ft (b) $\Omega = mg/(\pi \rho U D)$ (c) $\Omega = 12.3$ rpm

Chapter 11

11.1 $\dot{m} = 0$
11.3 $\Gamma = 0$
11.5 (a) $[Q] = L^2 T^{-1}$ (b) $[\Gamma] = L^2 T^{-1}$ (c) $\mathcal{D} = L^3 T^{-1}$
11.9 (a) $u = -2By$, $v = -2Ax$, Stagnation point at $x = y = 0$ (b) $B = A$
11.11 (a) $u_r = 3Qr^2 \cos 3\theta$, $u_\theta = -3Qr^2 \sin 3\theta$, Stagnation point at $r = 0$ (b) $Q = \frac{1}{4}$
(c) $x = \pm\sqrt{(y^2 + 4\psi/y)/3}$
11.13 $\phi = Ar^n \cos n\theta$, $F(z) = Az^n$
11.15 (a) $\phi = U(r + R^2/r)\cos\theta$, $\psi = U(r - R^2/r)\sin\theta$, Flow past a cylinder of radius R
(b) $F(z) = U(z + R^2/z) - i\Gamma/(2\pi)\ell n z$
11.17 $y = \pm 2A\sqrt{x + A^2}$, $A =$ constant
11.19 Stagnation point at $x = y = 0$, No
11.21 Stagnation point at $x = y = 0$, Yes
11.23 $\phi = U(x^2 + y^2 - 2z^2)/(2h)$, Yes
11.25 $\phi = U(x\cos\alpha + y\sin\alpha)$, $\psi = U(y\cos\alpha - x\sin\alpha)$
11.27 (a) $u_r = 2Sr\sin 2\theta$, $u_\theta = 2Sr\cos 2\theta$ (b) $\phi = Sr^2 \sin 2\theta$
(c) $x = y = 0$ (d) Plane at 45° to x axis
11.29 $\phi = Qr^n(\cos n\theta - \sin n\theta)$
11.31 $\phi = r^n \cos n\theta$
11.33 $p = p_\infty - \frac{1}{2}\rho Q^2/(2\pi r)^2$, $r_{min} = 10.4$ cm

11.35 $p = p_\infty - \frac{1}{2}\rho\mathcal{D}^2/(2\pi r)^2$, $r_{min} = 1.75$ ft
11.39 $r = r_o e^{Q\theta/\Gamma}$, $r_o = e^{-2\pi\psi/\Gamma}$
11.41 $x = -Q/(2\pi U)$, $y = \Gamma/(2\pi U)$
11.43 The thickness is Q/U
11.45 (a) $\psi = Ur\sin\theta - Q\theta/(2\pi)$, $\phi = Ur\cos\theta - Q\ell nr/(2\pi)$
(b) $p = p_\infty + \frac{1}{2}\rho[UQ/(\pi x) - Q^2/(2\pi x)^2]$ (c) $x = Q/(2\pi U)$
11.47 (c) $\mathbf{F} = -\rho QU\,\mathbf{i}$
11.49 (a) $Q = \sqrt{3}(\Gamma_a - \mathcal{D}/R)$
11.51 $\mathbf{F} = [2(p_i - p_\infty) + \frac{5}{3}\rho U^2]LR\,\mathbf{j}$
11.53 $D = \frac{8}{3}\rho U^2 R$
11.55 $\sin\theta_o = \sqrt{\frac{2}{3}}$
11.57 (a) $\mathbf{F} = \pi\rho\Omega D^2 H[V\sin\alpha\,\mathbf{i} - (U + V\cos\alpha)\,\mathbf{j}]$
(b) $F_x = \pi\rho\Omega D^2 H\sqrt{V^2 - U^2}$ (c) $F_x = 27$ tons
11.61 (a) $C_p = [1 - 4\sin^2\theta]\sin^2\omega t - 4St\cos\theta\cos\omega t$, where $St \equiv \omega R/U_o$
(b) $\overline{C}_p = \frac{1}{2}[1 - 4\sin^2\theta]$
11.63 (a) $C_p = 1 - 4\sin^2\theta - 4(R\dot{U}/U^2)\cos\theta$ (b) $L = 0$, $D = 2\pi\rho R^2\dot{U}$
11.65 $p = p_\infty - \rho\dot{Q}\theta/(2\pi) - \rho Q^2/(8\pi^2 r^2)$
11.67 (a) $\alpha = \sin^{-1}(0.5) = 30^o$ (b) $p_1 - p_2 = \frac{1}{2}\rho U^2 + 0.268\rho R\dot{U}$
11.69 $u(0,y)/U = 1.125, \infty, 1.260, 1.033$ at $y/R = 0, 4, 6, 10$
11.71 $L = \rho U\Gamma_a[1 - \Gamma_a/(4\pi Uh)]$, Lift will decrease due to the *ground effect*
11.73 $C_L = 1.10$
11.75 $2T_{max}/c = 12.6\%$, $C_{max}/c = 3.0\%$
11.77 (b) $C(x) = x\tan\alpha + \gamma_o x(1 - x/c)/(4U\cos\alpha)$
11.79 (b) $C(x) = x\tan\alpha + \gamma_o c[3x/c - 1 - 8(x/c - 1/2)^3]/(48U\cos\alpha)$
11.81 $A = -1/(12\Delta x)$, $B = 2/(3\Delta x)$, $C = 0$, $D = -2/(3\Delta x)$, $E = 1/(12\Delta x)$
11.83 $\phi_{i,j} = \frac{1}{2}[\phi_{i,j+1} + \phi_{i,j-1} + \beta^2(\phi_{i+1,j} + \phi_{i-1,j})]/(1 + \beta^2)$
11.85 $\phi_\infty = 2.95049, 3.05901, 3.07902, 3051901$ at $j = 1, 5, 9, 13$
11.87 $E_h \approx \phi_h - \phi_{2h}$
11.89 $\lambda_{opt} = 0.99$
11.91 (a) $p = 0.90$ (b) $GCI = 1.96\%$ (c) Richardson extrapolation: 0.45%

Chapter 12

12.1 $\ell_{mfp} = 1.5\nu/a = 648$ Å
12.3 $Kn = \ell_{mfp}/y$, Newtonian for $y \gg \ell_{mfp}$
12.5 (a) $\psi(r,\theta) = -\frac{1}{2}\Omega r^2 + \text{constant}$ (b) Flow is rotational

12.7 $\boldsymbol{\omega} = \mathbf{0}$, $[\mathbf{S}] = \dfrac{2U}{L}\begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

12.9 (a) $f(y) = Cy + v_o,\ g(x) = Cx + u_o$, (b) $[\mathbf{S}] = C\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

12.11 $\boldsymbol{\omega} = \mathbf{0}, \quad [\mathbf{S}] = \dfrac{U}{H}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{bmatrix}$

12.13 $\boldsymbol{\omega} = -\dfrac{3U}{a}\mathbf{k}, \quad [\mathbf{S}] = \dfrac{U}{2a}\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

12.15 $\boldsymbol{\omega} = \dfrac{2Uz}{L^2}(-\mathbf{i} + \mathbf{j}), \quad [\mathbf{S}] = \dfrac{U}{L^2}\begin{bmatrix} 2x & 0 & -z \\ 0 & 2y & -x \\ -z & -z & -2(x+y) \end{bmatrix}$

12.17 $[\mathbf{S}] = \dfrac{3Ur^2}{a^3}\begin{bmatrix} \cos 4\theta & -\sin 4\theta & 0 \\ -\sin 4\theta & -\cos 4\theta & 0 \\ 0 & 0 & 0 \end{bmatrix}$

12.19 $[\boldsymbol{\tau}] = \dfrac{\mu U}{L^3}\begin{bmatrix} 6y^2 & (6xy + 5z^2) & 0 \\ (6xy + 5z^2) & 0 & (10xz - 6yz) \\ 0 & (10xz - 6yz) & -6y^2 \end{bmatrix}$

$$\nabla \cdot [\boldsymbol{\tau}] = \frac{2\mu U}{L^3}(3x\,\mathbf{i} + 5x\,\mathbf{j} - 3z\,\mathbf{k})$$

12.21 $[\boldsymbol{\tau}] = \dfrac{\mu U}{H^2}\begin{bmatrix} 2y & x & 0 \\ x & -4y & z \\ 0 & z & 2y \end{bmatrix}, \quad \nabla \cdot [\boldsymbol{\tau}] = -\dfrac{2\mu U}{H^2}\,\mathbf{j}$

12.23 $tr[\boldsymbol{\tau}] = (2\mu + 3\zeta)\nabla \cdot \mathbf{u}$
12.25 $u = 4Ay, \quad v = -Ax$
12.27 (a) $\tau_{zz}, \ \nabla \cdot \mathbf{u} = K\cos\theta/r^2$
(b) $\tau_{rr}, \ u_r = K\cos\theta/r + f'(\theta)$
(c) $u_\theta = K\sin\theta/r - f(\theta) + g(r)$
(d) $f''(\theta) + rg'(r) + f(\theta) - g(r) = 0$
(e) $f''(\theta) - rg'(r) + f(\theta) - g(r) = 0$
(f) $f(\theta) = A\cos\theta + B\sin\theta + C, \quad g(r) = C \quad (A, B, C$ are constants)
(g) $u_r = (K/r)(1 + r/R)\cos\theta, \ u_\theta = (K/r)(1 - r/R)\sin\theta$
12.29 (b) The torque is negligible since p acts at the center of each side and has zero moment arm as $\Delta x, \Delta y \to 0$
(c) The angular momentum is $\leq \rho \max\{\Delta x, \Delta y\} \max|\mathbf{u}| \to 0$ as $\Delta x, \Delta y \to 0$
(d) $\tau_{xy} = \tau_{yx}$
12.33 $p = p_o - \frac{1}{2}\rho U^2(r/R)^2[1 - (r/R)^2] + \frac{1}{2}\rho U^2(z/R)^2[2 - (z/R)^2] - 10\mu Uz/R^2$
$p(0,0) = p_o$, Infinite pressure for $r, |z| \to \infty$

12.35 $p(r) = p_\infty - \rho(Q^2 + \Gamma^2)/(8\pi^2 r^2)$, Negative pressure for $r \to 0$
12.37 (b) $\nu f'' + f^2 + 4\nu f = \text{constant}$
12.39 (b) $f(r) = Cr + D/r$ (C and D are constants)
12.41 (a) $[H] = L^2$
12.45 $N_{CFL} = \frac{1}{2} Re_{\Delta x}$
12.49 $G = \cos\theta - i\lambda \sin\theta$, where $\lambda = U\Delta t/\Delta x$ and $\theta = \kappa\Delta x$
Conditionally stable, $\Delta t \le \Delta x/U$
12.51 $G = [1 + \lambda e^{-i\theta}]/[1 + \lambda e^{i\theta}]$, where $\lambda = \frac{1}{2} U\Delta t/\Delta x$ and $\theta = \kappa\Delta x$
Neutrally stable for all Δt
12.53 (a) $N = 101$: $(\tilde{N}_{CFL})_{max} = 1.053$, $N = 201$: $(\tilde{N}_{CFL})_{max} = 1.025$,
$N = 401$: $(\tilde{N}_{CFL})_{max} = 1.012$
12.55 (b) $F = -\partial\overline{u}/\partial\overline{y}$ (c) $\Delta\overline{t}_{max} = \frac{1}{2}(\Delta\overline{y})^2$ (d) 654 channel heights

Chapter 13

13.1 $\psi = \frac{1}{2} Uh[1 - \chi(1 - \frac{2}{3} y/h)](y/h)^2$
13.3 $\psi = \frac{1}{2} u_m R[1 - \frac{1}{2}(r/R)^2](r/R)$
13.5 $\psi = Uy\,\text{erfc}(\eta) + 2U\sqrt{\nu t/\pi}\,(1 - e^{-\eta^2})$, $\eta \equiv \frac{1}{2} y/\sqrt{\nu t}$

13.7 $\boldsymbol{\omega} = 2u_m r/R^2 \mathbf{e}_\theta$, $[\mathbf{S}] = -\dfrac{u_m r}{R^2}\begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}$, $[\boldsymbol{\tau}] = -2\dfrac{\mu u_m r}{R^2}\begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}$

13.9 $[\mathbf{S}] = -\dfrac{1}{2} s(\eta, t)\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$, $[\boldsymbol{\tau}] = -\mu s(\eta, t)\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

where $s(\eta, t) \equiv u_o\sqrt{\Omega/\nu}\, e^{-\eta} \sin(\Omega t - \eta + \pi/4)$, $\eta \equiv y\sqrt{\Omega/(2\nu)}$

13.11 $\dot{m} = \frac{1}{2}\rho Uh$, $\dot{P} = \frac{1}{3}\rho U^2 h$, $\dot{E} = \frac{1}{8}\rho U^3 h$
13.13 $y = (\chi - 1)h/(2\chi)$, $\chi = 3$
13.17 (a) $P = [6\mu U\ell^2/(h_0 - h_1)^2][\ell n(h_0/h_1) - 2(h_0 - h_1)/(h_0 + h_1)]$
(b) $F = [\mu U\ell/(h_1 - h_0)]\ell n(h_1/h_0)$
(c) $\mu_{\text{eff}} = 4.125 h_1/\ell$
(d) $\mu_{\text{eff}}/\mu_{\text{sliding}} = 0.02$
13.19 $(u_m)_{max} = 0.5$ m/sec, $\ell_e = 7.2$ m
13.21 12 cm
13.27 (a) $u = u_r = 0, u_\theta = \Omega R$ at $r = R$ and $\partial u/\partial r = 0, u_r = u_\theta = 0$ at $r = 0$
(b) $u_r(r) = 0$
(c) $u(r) = u_m[1 - (r/R)^2]$, $u_\theta(r) = \Omega r$ where $u_m = -R^2 \partial p/\partial x/(4\mu)$
(d) $p(x, r) - p(x, 0) = \frac{1}{2}\rho\,\Omega^2 r^2$

13.29 (a) $u_r = 0, u_\theta = \Omega R$ at $r = R$ and $u_r \to 0, u_\theta \to 0$ as $r \to \infty$
(b) $u_r(r) = 0,\ u_\theta(r) = \Omega R^2/r$
(c) $\Gamma = 2\pi\Omega R^2,\ u_\theta(r) = \Gamma/(2\pi r)$, Potential vortex
(d) $p(r) = p_\infty - \frac{1}{2}\rho\,\Omega^2 R^4/r^2,\ \Omega < \sqrt{2p_\infty/(\rho R^2)}$

13.31 (b) $u(z) = U[1 - e^{-\lambda z}\cos\lambda z],\ v(z) = Ue^{-\lambda z}\sin\lambda z,\ \lambda \equiv \sqrt{\Omega/\nu}$

13.33 (a) $v(y,t) = v_w$
(b) $\partial u/\partial t + v\partial u/\partial y = \nu\partial^2 u/\partial y^2$
(c) $v\sqrt{t/\nu} = \text{constant}$
(d) No similarity solution exists

13.37 (b) $F(0) = 0,\ G(0) = 1,\ H(0) = 0,\ P(0) = 0,\ F(\zeta) \to 0, G(\zeta) \to 0$ as $\zeta \to \infty$

13.39 $r = 1.62r_o$

13.43 Yes

13.45 Matrices $[A]$ and $[C]$ are diagonally dominant. Matrix $[B]$ is not diagonally dominant.

13.47 $\mathbf{x} = \begin{Bmatrix} 1.0000 \\ 1.0423 \\ 1.1128 \\ 1.2113 \\ 1.0000 \end{Bmatrix}$

13.49 (a) $\phi(y) = (e^y - e^{-y})/(e - e^{-1})$

(b) $$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 1 & -33/16 & 1 & 0 & 0 \\ 0 & 1 & -33/16 & 1 & 0 \\ 0 & 0 & 1 & -33/16 & 1 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{Bmatrix} \phi_1 \\ \phi_2 \\ \phi_3 \\ \phi_4 \\ \phi_5 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{Bmatrix}$$

(c) $$\{\boldsymbol{\phi}\} = \begin{Bmatrix} 0 \\ 0.215114 \\ 0.443674 \\ 0.699963 \\ 1 \end{Bmatrix} \quad \text{and} \quad \{\boldsymbol{\phi_{exact}}\} = \begin{Bmatrix} 0 \\ 0.214952 \\ 0.443409 \\ 0.699724 \\ 1 \end{Bmatrix}$$

13.51 (c) No limit on Δt
(d) $1.98 \le p \le 2.01$
$GCI \le 0.024\%$

13.53 (a) $P(\zeta) = -2F(\zeta) - \frac{1}{2}H^2(\zeta)$
(c) $F(\zeta) > 0$
(d) $1.86 \le p \le 1.91$
$GCI \le 0.045\%$

Chapter 14

14.1 (a) $Re_L = 1.0 \cdot 10^6$ (b) $Re_L = 2.0 \cdot 10^5$
14.3 Air: $U = 16.20$ ft/sec, H_2O: $U = 1.08$ ft/sec
14.5 $\delta = 103$ mm
14.7 $x_{tot} = 296$ ft (6.6% smaller)
14.9 (a) $d^2u/d\eta^2 + (1+\epsilon)du/d\eta + \epsilon u = 0$
(b) $u_{\text{inner}}(\eta) = C(1 - e^{-\eta})$
(c) $C = e$
14.13 $u(y) = U(1 - e^{-v_w y/\nu})$, $v_w > 0$
14.15 $u\partial\omega/\partial x + v\partial\omega/\partial y = \nu\partial^2\omega/\partial y^2$
14.17 $\delta/x = 0.37Re_x^{-1/5}$, $c_f = 0.0577Re_x^{-1/5}$, $D = 0.072\rho U^2 bLRe_L^{-1/5}$
14.19 (a) $c_f = 2\nu/(U\delta)$, $\delta^* = \frac{1}{2}\delta$, $\theta = \frac{1}{6}\delta$, $H = 3.00$
(b) $\delta = 3.46xRe_x^{-1/2}$
14.21 (a) $c_f = 3\nu/(U\delta)$, $\delta^* = \frac{3}{8}\delta$, $\theta = \frac{39}{280}\delta$, $H = 2.69$
(b) $\delta = 4.64xRe_x^{-1/2}$
14.23 (a) $c_f = 4\nu/(U\delta)$, $\delta^* = \frac{3}{10}\delta$, $\theta = \frac{37}{315}\delta$, $H = 2.55$
(b) $\delta = 5.84xRe_x^{-1/2}$
14.25 (b) $d\delta/dx = (6 + 10\beta_T)/Re_\delta$
14.27 (a) $F(K) = c_f Re_\theta - 2(2+H)K$
(c) $\theta = 0.671\sqrt{\nu x/U}$, 1.1% larger than Blasius
14.33 $\theta_E = \delta/4$, $\delta = 4\sqrt{\nu x/u_e}$, 20% smaller than Blasius
14.35 $\delta = 10L\sqrt{\nu/(15U_o x)}$, $c_f = 3.87Re_x^{-1/2}$, 6% smaller than the exact Falkner-Skan c_f
14.37 $x = 1$ ft: $\delta = 0.21$ in, $c_f = 2.37 \cdot 10^{-3}$, $x = 5$ ft: $\delta = 0.48$ in, $c_f = 1.06 \cdot 10^{-3}$
14.39 $Re_L = 4.94 \cdot 10^4$, $\delta \approx 0.27$ in
14.43 (a) $G(0) = 0$, $G'(\eta) \to 1$ as $\eta \to \infty$, $G'(\eta) \to U_2/U_1$ as $\eta \to -\infty$
(b) $d^3g/d\eta^3 + \eta d^2g/d\eta^2 \approx 0$
$g(0) = 0$, $g'(\eta) \to 0$ as $\eta \to \infty$, $g'(\eta) \to (U_2 - U_1)/U_1$ as $\eta \to -\infty$
(c) $u(x,y) \approx U_1 - (U_1 - U_2)\,\text{erfc}\left(\frac{1}{2}y\sqrt{U_1/\{\nu x\}}\right)$
14.45 $\ell_m = 0.01HRe_H = \ell_e/6$
14.49 $Re_L = 9.78 \cdot 10^6$, $\delta \approx 3.2$ in
14.51 (a) 11%, (b) 4.2%, (c) 4.7%, (d) 0.9%
14.53 (a) $x_t = 3.24$ ft, $\Delta x = 0.64$ ft, $(c_f)_t = 0.00047$, $(c_f)_f = 0.00305$
(b) $x_t = 2.16$ in, $\Delta x = 0.68$ in, $(c_f)_t = 0.00094$, $(c_f)_f = 0.00395$
14.55 $C_D = 0.144Re_L^{-1/5}$, $C_D/C_{D\ell} = 3.42, 6.81, 13.60$ for $Re_L = 10^5, 10^6, 10^7$
14.57 Mooring Line: 110 Hz, audible, Piling: 2.2 Hz, not audible
14.59 (b) $Re_x = 2.8 \cdot 10^7$: $h_\delta/\delta_{sl} \approx 14$, $Re_x = 5.0 \cdot 10^8$: $h_\delta/\delta_{sl} \approx 10$
14.61 $c_f = 0.00251$, which is 4.6% less than the Kármán-Schoenherr value
14.63 Copper: $U_{max} = 1908$ m/sec
Galvanized Iron: $U_{max} = 11.4$ m/sec
Concrete: $U_{max} = 0.89$ m/sec
14.65 $u_\tau = 0.241$ m/sec, $k_s^+ = 4519$
14.67 $f(0) = 0$, $f'(\eta) \to 1$ as $\eta \to \infty$, $f'(\eta) \to 0$ as $\eta \to -\infty$, $n(x) = U_1 x$

14.69 $\tau_t \propto y^4$ as $y^+ \to 0$
14.71 Smooth: $c_f = 0.00264$, $C_D = 0.00330$
Rough: $c_f = 0.00446$, $C_D = 0.00573$
c_f is 69% larger, C_D is 74% larger
14.73 (a) $(\nu + \nu_t)d\overline{u}/dy - v_w\overline{u} = u_\tau^2$
(b) $\kappa^2 y^2 (d\overline{u}/dy)^2 = u_\tau^2 + v_w\overline{u}$
14.75 (a) $c_f = 0.00250$ (b) $c_f = 0.00642$
14.81 (a) $2G''(0)\Theta - 2(2 + \Delta^*/\Theta)K \approx 0.450 - 6K, \;\; K = \beta\Theta^2$
14.85 Baldwin-Barth model: The boundary layer separates
k-ω model: Computed c_f is within 16% of measured c_f
k-ϵ models: Computed c_f is nearly triple measured c_f at $s = 17.833$ ft
14.87 k-ω model: Computed c_f is within 8% of measured c_f
Spalart-Allmaras model: Computed c_f is within 14% of measured c_f
k-ϵ models: Computed c_f is at least 48% larger than measured c_f at $s = 6.167$ ft
14.89 (b) k-ω model: $\theta_{sep} = 83.1^\circ$, Boundary layer fails to separate
for all k-ϵ models and for the Spalart-Allmaras model

Chapter 15

15.3 (b) $x \to -\infty$: $u \to u_1, h \to h_1, \rho \to \rho_1$ and $x \to +\infty$: $u \to u_2, h \to h_2$
15.7 (a) $H(0) = 1$, $H(\eta) \to 0$ as $\eta \to \infty$
(b) $H(\eta) = \text{erfc}(\sqrt{Pr}\,\eta)$
15.9 (b) $a = \sqrt{gh}, \;\; \gamma = 2$
15.11 $L_{max} = 1.89$ m, $T_e = 17^\circ$ C
15.13 $\overline{f} = 0.016, \;\; M_e = 1.18$
15.15 (a) $L_i^* = 125$ cm (b) Yes
15.17 $M_e = 1.23, \;\; p_e = 42.7$ psi, $T_e = 600^\circ$ F
15.19 $\overline{f} = 0.012, \;\; M_e = 1.29$
15.23 $q = 13265$ Btu/slug, $p_e = 13.4$ psi
15.25 Air: $q = 2.45 \cdot 10^4$ J/kg, $T_e = 415$ K
H_2: $q = 6.50 \cdot 10^6$ J/kg, $T_e = 681$ K
15.27 $M = 0.5$: $q = 2.92 \cdot 10^5$ J/kg
$M = 0.7$: $q = 4.76 \cdot 10^5$ J/kg
$M = 0.9$: $q = 5.47 \cdot 10^5$ J/kg
15.29 $q = -3543$ Btu/slug, $M_e = 2.56$
15.31 $q = 9.28 \cdot 10^5$ J/kg, $p^* = 89.6$ kPa
15.35 $\theta = 9.1^\circ$
15.37 $p_{t2}/p_{t1} = 0.9919$
15.39 $p_A = 36.1$ psi, $p_B = 241$ psi
15.41 (a) $M = 4.39$, $A/A^* = 15.14$
(b) $M = 3.57$, $A/A^* = 7.27$
(c) $M = 1.59$, $A/A^* = 1.24$
15.43 $p_{t2} = 103$ kPa

15.47 Air: $p_2/p_1 = 16.33$, He: $p_2/p_1 = 21.51$
15.49 $M_3 = 1.02$, $p_3 = 727$ kPa, $\beta_r = 42.8^o$
15.51 $\Phi = 27.1^o$
15.55 $\nu(M) \to \frac{1}{2}\pi[\sqrt{(\gamma+1)/(\gamma-1)} - 1]$ as $M \to \infty$
CO_2: Turn the flow 165^o, Air: Turn the flow 130^o, He: Turn the flow 90^o
15.57 Forward: 19.5^o, Rearward: 5.2^o
15.59 $M_1 = 2.35$, $\theta = 14.23^o$
15.61 $M_3 = 3.73$, $p_3 = 0.74$ atm
15.63 $M_3 = 1.75$, $p_3 = 55$ kPa
15.65 $x/L = 2.66$, $y/L = 1.19$
15.67 $x/L = 3.38$, $y/L = 1.75$
15.69 $\mu = 4.681 \cdot 10^{-9} T^{0.7}$
15.73 $C_{Df} = 2.464/\sqrt{Re_L}$
$Re_L = 10^4$: $C_{Df}/C_{Dw} = 0.122$
$Re_L = 10^5$: $C_{Df}/C_{Dw} = 0.039$
$Re_L = 10^6$: $C_{Df}/C_{Dw} = 0.012$
15.75 (a) $\delta_o = \sqrt{12 L_w \mu_e x/(\rho_e u_e)}$
15.77 $\tilde{u}/u_\tau = 26.51 \sin([u^*/u_\tau]/26.51)$ and the difference between $\tilde{u}/u_\tau$ and u^*/u_τ increases from 2.6% to 9.8% as y^+ increases from 10 to 500.
15.79 $\nu_a = \lambda a \Delta x/2$, $\mathcal{D}_a = (4\lambda^2 - 3\lambda - 1)a(\Delta x)^2/6$ where $\lambda \equiv a\Delta t/\Delta x$
15.81 $\nu_a = 0$, $\mathcal{D}_a = -(\lambda^2 + 2)a(\Delta x)^2/12$ where $\lambda \equiv a\Delta t/\Delta x$
15.85 k-ω model: Computed c_f is within 0.6% of measured c_f
k-ϵ model: Computed c_f is 67% larger than measured c_f
Spalart-Allmaras model: Computed c_f is within 1.2% of measured c_f
15.87 k-ω model: Computed c_f is within 3% of measured c_f
k-ϵ model: Computed c_f is twice measured c_f
Spalart-Allmaras model: Computed c_f is within 18% of measured c_f